Wave Function of a Supersymmetric FRW Model with Vector Fields

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Abstract

A specific Friedmann-Roberstson-Walker (FRW) model derived from the theory of N=1 supergravity with gauged supermatter is considered in this paper. The supermatter content is restricted to a vector supermultiplet. The corresponding Lorentz and supersymmetry quantum constraints are then derived. Non-trivial solutions are subsquently found. A no-boundary solution is identified while another state may be interpreted as a wormhole solution.

1   Introduction

Research in supersymmetric quantum cosmology using canonical methods started about 20 years or so [1]-[3]. Since then, many other papers have appeared in the literature [4]-[28]. Recent reviews on the subject of canonical quantum supergravity can be found in refs. [29, 30].

An important feature of N=1 supergravity is that it constitutes a "square-root" of gravity [1]. In fact, the Lorentz and supersymmetry constraints present in the canonical formulation of the theory induce a set of coupled *first-order* differential equations which the quantum states ought to satisfy. Such physical states will automatically satisfy the usual Hamiltonian and momentum constraints as a consequence of the supersymmetry algebra [1]-[3]. Hence, canonical quantization methods may provide us with supplementary and attractive insights as far as quantum supergravity theories are concerned. In particular, when dealing with (ultraviolet) divergences in quantum cosmology and gravity [31] and also removing Planckian masses induced by wormholes [4, 32].

Important results for Bianchi class-A models were recently achieved within pure N=1 supergravity. On the one hand, the Hartle-Hawking (no-boundary) [33] and wormhole (Hawking-Page) [34] states were found in the *same* spectrum of solutions [12, 13]. This result improved previous attempts [5]-[11] where only one of these states could be found, depending on the homogeneity conditions imposed on the gravitino [7]. The reason was an overly restricted ansatz for the wave function of the universe, Ψ. More precisely, gravitational degrees of freedom have not been properly taken into account in the expansion of Ψ in Lorentz invariant fermionic sectors (see ref. [12, 13, 30] for more details). Another undesirable consequence of the ansatz used in [5]-[11]

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was the following: Bianchi class-A models were found to have *no* physical states but the trivial one, $\Psi = 0$, when a cosmological constant was included [14]-[16]. However, an extension of the method in [12, 13] did led in ref. [17] to solutions of the form of exponentials of the Chern-Simons functional.

The introduction of supermatter [35] was contemplated in ref. [18]–[26]. A scalar supermultiplet, constituted by complex scalar fields, $\phi, \bar{\phi}$ and their spin$-\frac{1}{2}$ partners, $\chi_A, \bar{\chi}_{A'}$ was considered in ref. [4, 9, 19]–[26]. A vector supermultiplet, formed by a vector field $A_\mu^{(a)}$ and its supersymmetric partner, was added in ref. [21, 22]. FRW models were investigated in ref. [4, 9, 19]–[24] while a Bianchi IX model was analysed in ref. [25, 26]. Bianchi class-A models with Maxwell fields within N=2 supergravity were considered in ref. [27, 28]. The main results can be summarized as follows:

- A wormhole solution was obtained in ref. [4], but *not* in ref. [20]. We emphasize that the more general theory of N=1 supergravity with gauged supermatter [35] was employed in ref. [20]. The reason for the discrepancy between [4, 20] was analysed in ref. [23, 24] and related to the type of Lagrange multipliers and fermionic derivative ordering that were used.

- As far as a Hartle-Hawking state is concerned, some of the solutions present in [4, 9, 19]–[24] bear some of the properties corresponding to the no-boundary proposal [33]. Unfortunately, the supersymmetry constraints were *not* sufficient in determining the dependence of Ψ with respect to the scalar field (cf. ref. [23, 24] for more details).

- The results found within the more general supermatter content used in ref. [21, 22] were disapointing: the only allowed physical state was $\Psi = 0$.

- Exponentials of the N=2 supersymmetric Chern-Simons functional were the *only* type of solutions found in [27, 28] for the Bianchi class-A models.

It is certainly of interest to investigate further some of these issues. On the one hand, the apparent absence of wormhole solutions [23, 24] and the difficulty to obtain an adequate Hartle-Hawking solution when scalar supermultiplets are considered. On the other hand, why non-trivial physical states are not permitted when all possible matter fields are present [21, 22]. In addition, how can obtain other types of solutions different from the ones described in ref.[27, 28]?

In this paper we will consider a closed FRW model within the theory of N=1 supergravity with supermatter restricted to a vector supermultiplet. Out purpose is twofold. First, to find and subsequently analyse possible solutions of the quantum constraints present in our model. Finally, we aim in providing a new perspective on the issues concerning [21, 22].

In section 2 we will describe our field variables and then derive the corresponding Lorentz and supersymmetry constraints. *Non-trivial* solutions in different fermionic sectors are subsequently obtained. We identify a *component* of the Hartle-Hawking (no-boundary) solution [33, 36]. Another solution could be interpreted as a quantum wormhole state [34] (see also ref. [38]). We stress that the Hartle-Hawking solution found here is part of the set of solutions also present in ref. [36], where a *non*-supersymmetric FRW minisuperspace with Yang-Mills fields was instead considered. Since N=1 supergravity is a square-root of gravity, our results are thus particularly interesting and consistent with what should be expected (see also ref. [1, 8, 9, 29, 30]). Our discussions and conclusions are present in section 3. Finally, we close this paper with an appendix where we describe and discuss the choice of configuration for the field variables employed in this paper.

2 Canonical Formulation and Quantization

2.1 Field Variables

The action for our model is obtained from the more general theory of N=1 supergravity with gauged supermatter present in ref. [35] (see eq. (25.12)). We choose to put all scalar fields and corresponding supersymmetric partners equal to zero[1], i.e., $\bar{\phi}^{I*} = \phi^I = 0$, $\chi^I_A = \bar{\chi}^{I*}_{A'} = 0$. Similar systems with Yang-Mills fields coupled to N=1 supergravity can also be found in ref. [43].

Our field variables will be constituted by a tetrad, $e^{AA'}_\mu$ (in two spinor component notation[2]), gravitino fields, $\psi^A_\mu, \bar{\psi}^{A'}_\nu$, (where a bar denotes Hermitian conjugation), a vector field, $A^{(a)}_\mu$, (where (a) is a group index) and the the corresponding spin-$\frac{1}{2}$ partners, $\lambda^{(a)}_A, \bar{\lambda}^{(a)}_{A'}$.

The restriction of this theory to a closed FRW model requires the introduction of adequate ansätze for the fields mentioned above. These are discussed in more detail in the Appendix. The tetrad can be written as

$$e_{a\mu} = \begin{pmatrix} N(\tau) & 0 \\ 0 & a(\tau)E_{\hat{a}i} \end{pmatrix} \ , \ e^{a\mu} = \begin{pmatrix} N(\tau)^{-1} & 0 \\ 0 & a(\tau)^{-1}E^{\hat{a}i} \end{pmatrix} \ , \tag{1}$$

where $\hat{a}$ and i run from 1 to 3. $E_{\hat{a}i}$ is a basis of left-invariant 1-forms on the unit S^3.

As far as the gravitino fields are concerned, the Lagrange multipliers $\psi^A{}_0$ and $\bar{\psi}^{A'}{}_0$ are taken to be functions of time only. We further include

$$\psi^A{}_i = e^{AA'}{}_i\bar{\psi}_{A'} \ , \ \bar{\psi}^{A'}{}_i = e^{AA'}{}_i\psi_A \ , \tag{2}$$

where the new spinors ψ_A and $\bar{\psi}_{A'}$ are functions of time only. This means we truncate the general decomposition $\psi^A{}_{BB'} = e_{BB'}{}^i\psi^A{}_i$,

$$\psi_{ABB'} = -2n^C{}_{B'}\gamma_{ABC} + \frac{2}{3}\left(\beta_A n_{BB'} + \beta_B n_{AB'}\right) - 2\varepsilon_{AB}n^C{}_{B'}\beta_C \ , \tag{3}$$

where $\gamma_{ABC} = \gamma_{(ABC)}$, at spin$-\frac{1}{2}$ mode level. I.e., $\beta^A = \frac{3}{4}n^{AA'}\overline{\psi}_{A'} \sim \overline{\psi}^A$.

Concerning the matter fields, we will consider here the choice employed in ref. [36]-[40] for the vector field $A^{(a)}_\mu$ (see also ref. [21, 22]). This configuration is the simplest one that allows vector fields to be present *consistently* in a closed FRW geometry. Our spin-1 field configuration is taken to be:

$$\mathbf{A}_\mu(t) \ \omega^\mu = \left(\frac{f(t)}{4}\varepsilon_{(a)i(b)}\mathcal{T}^{(a)(b)}\right)\omega^i \ . \tag{4}$$

Here $\{\omega^\mu\}$ denotes $\{\omega^\mu\} = \{dt, \omega^i\}$, where $\omega^i = \hat{E}^i{}_{\hat{c}}dx^{\hat{c}}$ $(i, \hat{c} = 1, 2, 3)$ are left-invariant one-forms on S^3, and $\mathcal{T}_{(a)(b)}$ are the generators of an internal group of transformations. For simplicity, we will restrict ourselves to the case of a $SU(2)$ group, with $\tau_{(a)} = -\frac{1}{2}\epsilon_{(a)(b)(c)}\mathcal{T}_{(b)(c)}$ being the usual $SU(2)$ matrices. The ansatz (4) implies $A^{(a)}_\mu$ to be paramatrized by a single scalar function $f(t)$. FRW cosmologies with this ansatz are totally equivalent to a FRW minisuperspace with an effective conformally coupled scalar field, but with a quartic potential instead of a quadratic one. The choice (4) simplifies   considerably any analysis of the Hamiltonian constraints (see [36, 37, 41]) and this constituted another compelling argument to use it.

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[1] An important consequence of not having scalar fields and their fermionic partners is that the Killing potentials $D^{(a)}$ and all related quantitites are now absent [30, 35].

[2] I.e., $e^{AA'}_\mu = e^a_\mu\sigma^{AA'}_a$ where $a, \mu = 0, 1, 2, 3$ are, respectively space-time and Lorentz indices, while $A = 0, 1, A' = 0', 1'$ are spinor indices and $\sigma^a_{AA'}$ are the Infeld-van de Warden translation symbols – cf., e.g., ref. [3].

As fermionic partner for $A_\mu^{(a)}$ we will use the more general choice

$$\lambda_A^{(a)} = \lambda_A^{(a)}(t) \ . \tag{5}$$

2.2 Quantum constraints and solutions

We now proceed towards obtaining solutions of the quantum Lorentz and supersymmetry constraints (cf. ref. [30] and references therein for further examples with other type of fields). To achieve this we need first to integrate the Lagrangian of N=1 supergravity with only vector multiplets (see ref. [35, 43]) over the angular variables of S^3. In this process we will use the configurations decribed in the previous subsection for the field variables. The next step consists in identifying our minisuperspace coordinates and canonical conjugate momenta. The presence of fermionic fields leads to second-class constraints (see e.g. ref [44]) and hence employing Dirac instead of Poisson brackets [2, 9]. According to the guidelines described in [22] we also redefine the fermionic fields, $\psi_A(t)$ and $\lambda_A^{(a)}(t)$ in order to simplify the Dirac brackets.

For the ψ_A-field we introduce,

$$\hat{\psi}_A = \frac{\sqrt{3}}{2^{\frac{1}{4}}} \sigma a^{\frac{3}{2}} \psi_A \ , \ \hat{\bar{\psi}}_{A'} = \frac{\sqrt{3}}{2^{\frac{1}{4}}} \sigma a^{\frac{3}{2}} \bar{\psi}_{A'} \ , \tag{6}$$

where the conjugate momenta are

$$\pi_{\hat{\psi}_A} = i n_{AA'} \hat{\bar{\psi}}^{A'} \ , \ \pi_{\hat{\bar{\psi}}_{A'}} = i n_{AA'} \hat{\psi}^A \ . \tag{7}$$

The Dirac brackets then become

$$[\hat{\psi}_A, \hat{\bar{\psi}}_{A'}]_D = i n_{AA'} \ . \tag{8}$$

Similarly for the $\lambda_A^{(a)}$ field

$$\hat{\lambda}_A^{(a)} = \frac{\sigma a^{\frac{3}{2}}}{2^{\frac{1}{4}}} \lambda_A^{(a)}, \ \hat{\bar{\lambda}}_{A'}^{(a)} = \frac{\sigma a^{\frac{3}{2}}}{2^{\frac{1}{4}}} \bar{\lambda}_{A'}^{(a)} \ , \tag{9}$$

giving

$$\pi_{\hat{\lambda}_A^{(a)}} = -i n_{AA'} \hat{\bar{\lambda}}^{(a)A'} \ , \ \pi_{\hat{\bar{\lambda}}_{A'}^{(a)}} = -i n_{AA'} \hat{\lambda}^{(a)A} \ , \tag{10}$$

with

$$[\hat{\lambda}_A^{(a)}, \hat{\bar{\lambda}}_{A'}^{(a)}]_D = -i \delta^{ab} n_{AA'} \ . \tag{11}$$

Furthermore,

$$[a, \pi_a]_D = 1 \ , \ [f, \pi_f]_D = 1, \tag{12}$$

and the rest of the brackets are zero.

It is simpler to describe the theory using only (say) unprimed spinors, and, to this end, we define

$$\bar{\psi}_A = 2 n_A^{B'} \bar{\psi}_{B'} \ , \ \bar{\lambda}_A^{(a)} = 2 n_A^{B'} \bar{\lambda}_{B'}^{(a)} \ , \tag{13}$$

with which the new Dirac brackets are

$$[\psi_A, \bar{\psi}_B]_D = i \epsilon_{AB} \ , [\lambda_A^{(a)}, \bar{\lambda}_{A'}^{(a)}]_D = -i \delta^{ab} \epsilon_{AB} \ . \tag{14}$$

4

The rest of the brackets remain unchanged. Quantum mechanically, one replaces the Dirac brackets by anti-commutators if both arguments are odd (O) or commutators if otherwise (E):

$$[E_1, E_2] = i[E_1, E_2]_D \ , \ [O, E] = i[O, E]_D \ , \ \{O_1, O_2\} = i[O_1, O_2]_D \ . \tag{15}$$

Here, we take units with $\hbar = 1$. The only non-zero (anti-)commutator relations are:

$$\{\lambda_A^{(a)}, \lambda_B^{(b)}\} = \delta^{ab}\epsilon_{AB} \ , \ \{\psi_A, \bar{\psi}_B\} = -\epsilon_{AB} \ , \ [a, \pi_a] = [f, \pi_f] = i \ . \tag{16}$$

We chose $\left(\bar{\lambda}_{A'}^{(a)}, \psi_A, a, f\right)$ to be the coordinates of the configuration space, and $\left(\lambda_A^{(a)}, \bar{\psi}_A, \pi_a, \pi_f\right)$ to be the momentum operators in this representation. Hence

$$\lambda^a_A \to -\frac{\partial}{\partial\bar{\lambda}^{(a)A}} \ , \ \bar{\psi}_A \to \frac{\partial}{\partial\psi^A} \ , \ \pi_a \to \frac{\partial}{\partial a}, \ \pi_f \to -i\frac{\partial}{\partial f} \ . \tag{17}$$

Following the ordering used in ref. [4], we put all the fermionic derivatives in S_A on the right. In $\bar{S}_A$, all the fermonic derivatives are on the left. Implementing all these redefinitions, the supersymmetry constraints have the differential operator form

$$
\begin{aligned}
S_A \ = \ & -\frac{1}{2\sqrt{6}}a\psi_A\frac{\partial}{\partial a} - \sqrt{\frac{3}{2}}\sigma^2 a^2\psi_A \\
& -\frac{1}{8\sqrt{6}}\psi_B\psi^B\frac{\partial}{\partial\psi^A} - \frac{1}{4\sqrt{6}}\psi^C\bar{\lambda}_C^{(a)}\frac{\partial}{\partial\bar{\lambda}^{(a)A}} \\
& +\frac{1}{3\sqrt{6}}\sigma^a_{AB'}\sigma^{bCC'}n_D^{B'}n_{C'}^B\bar{\lambda}^{(a)D}\psi_C\frac{\partial}{\partial\bar{\lambda}^{(b)B}} \\
& +\frac{1}{6\sqrt{6}}\sigma^a_{AB'}\sigma^{bBA'}n_D^{B'}n_{A'}^E\bar{\lambda}^{(a)D}\bar{\lambda}_B^{(b)}\frac{\partial}{\partial\psi^E} \\
& -\frac{1}{2\sqrt{6}}\psi_A\bar{\lambda}^{(a)C}\frac{\partial}{\partial\bar{\lambda}^{(a)C}} + \frac{3}{8\sqrt{6}}\bar{\lambda}^a_A\lambda^{(a)C}\frac{\partial}{\partial\psi^C} \\
& + \sigma^a_{AA'}n^{BA'}\bar{\lambda}_B^{(a)}\left(-\frac{\sqrt{2}}{3}\frac{\partial}{\partial f} + \frac{1}{8\sqrt{2}}(1 - (f-1)^2)\sigma^2\right)
\end{aligned}
\tag{18}
$$

and

$$
\begin{aligned}
\bar{S}_A \ = \ & \frac{1}{2\sqrt{6}}a\frac{\partial}{\partial a}\frac{\partial}{\partial\psi^A} - \sqrt{\frac{3}{2}}\sigma^2 a^2\frac{\partial}{\partial\psi^A} \\
& -\frac{1}{8\sqrt{6}}\varepsilon^{BC}\frac{\partial}{\partial\psi^B}\frac{\partial}{\partial\psi^C}\psi_A + \frac{1}{4\sqrt{6}}\varepsilon^{BC}\frac{\partial}{\partial\psi^B}\frac{\partial}{\partial\bar{\lambda}^{(a)C}}\bar{\lambda}_A^{(a)} \\
& +\frac{1}{3\sqrt{6}}\sigma^{aB}_{A'}\sigma^{bCC'}n_A^{A'}n_{C'}^D\frac{\partial}{\partial\psi^D}\frac{\partial}{\partial\bar{\lambda}^{(a)B}}\bar{\lambda}_C^{(b)} \\
& +\frac{1}{6\sqrt{6}}\sigma^{aB}_{A'}\sigma_D^{bB'}n_A^{A'}n_{B'}^C\frac{\partial}{\partial\bar{\lambda}^{(a)B}}\frac{\partial}{\partial\bar{\lambda}^{(b)C}}\psi^D \\
& +\frac{1}{2\sqrt{6}}\frac{\partial}{\partial\psi^A}\frac{\partial}{\partial\bar{\lambda}^{(a)B}}\bar{\lambda}^{(a)B} + \frac{3}{8\sqrt{6}}\frac{\partial}{\partial\bar{\lambda}^{(a)B}}\frac{\partial}{\partial\bar{\lambda}^{(a)A}}\psi^B \\
& + n_A^{A'}\sigma^{aB}_{A'}\left(\frac{2\sqrt{2}}{3}\frac{\partial}{\partial f} + \frac{1}{4\sqrt{2}}(1 - (f-1)^2)\sigma^2\right)\frac{\partial}{\partial\bar{\lambda}^{(a)B}}.
\end{aligned}
\tag{19}
$$

The Lorentz constraint has the form:

$$J_{AB} = \psi_{(A}\bar{\psi}^{B'}n_{B)B'} - \lambda^{(a)}_{(A}\bar{\lambda}^{(a)B'}n_{B)B'} = 0 \ . \tag{20}$$

The Lorentz constraint J_{AB} implies that a physical wave function should be a Lorentz scalar. We can easily see that the most general form of the wave function is

$$
\begin{aligned}
\Psi = A \ + \ & B\psi^C\psi_C + d_a\lambda^{(a)C}\psi_C + c_{ab}\bar{\lambda}^{(a)C}\bar{\lambda}^{(b)}_C + e_{ab}\bar{\lambda}^{(a)C}\bar{\lambda}^{(b)}_C\psi^D\psi_D \\
+ \ & c_{abc}\bar{\lambda}^{(a)C}\bar{\lambda}^{(b)}_C\bar{\lambda}^{(c)D}\psi_D + c_{abcd}\bar{\lambda}^{(a)C}\bar{\lambda}^{(b)}_C\bar{\lambda}^{(c)D}\bar{\lambda}^{(d)}_D + d_{abcd}\bar{\lambda}^{(a)C}\bar{\lambda}^{(b)}_C\bar{\lambda}^{(c)D}\bar{\lambda}^{(d)}_D\psi^E\psi_E \\
+ \ & \mu_1\bar{\lambda}^{(2)C}\bar{\lambda}^{(2)}_C\bar{\lambda}^{(3)D}\bar{\lambda}^{(3)}_D\bar{\lambda}^{(1)E}\psi_E \\
+ \ & \mu_2\bar{\lambda}^{(1)C}\bar{\lambda}^{(1)}_C\bar{\lambda}^{(3)D}\bar{\lambda}^{(3)}_D\bar{\lambda}^{(2)E}\psi_E + \mu_3\bar{\lambda}^{(1)C}\bar{\lambda}^{(1)}_C\bar{\lambda}^{(2)D}\bar{\lambda}^{(2)}_D\bar{\lambda}^{(3)E}\psi_E \\
+ \ & F\bar{\lambda}^{(1)C}\bar{\lambda}^{(1)}_C\bar{\lambda}^{(2)D}\bar{\lambda}^{(2)}_D\bar{\lambda}^{(3)E}\bar{\lambda}^{(3)}_E + G\bar{\lambda}^{(1)C}\bar{\lambda}^{(1)}_C\bar{\lambda}^{(2)D}\bar{\lambda}^{(2)}_D\bar{\lambda}^{(3)E}\bar{\lambda}^{(3)}_E\psi^F\psi_F \ . \tag{21}
\end{aligned}
$$

where $A, B,...,G$ are functions of a, f only. This Ansatz contains all allowed combinations of the fermionic fields and is the most general Lorentz invariant function we can write down.

The next step is to solve the supersymmetry constraints $S_A\Psi = 0$ and $\bar{S}_{A'}\Psi = 0$. Since the wave function (21) is of even order in fermionic variables and stops at order 8, the expressions $S_A\Psi = 0$ and $\bar{S}_A\Psi = 0$ will be of odd order in fermionic variables and stop at order 7. Since fermionic terms as $\psi^A, \chi^A, \psi^A\chi^C\chi_C$, etc, are linearly independent Grassmanian quantities, the action of the quantum operators (18), (19) on (21) (see also appendix B in ref. [35]) will produce ten equations from $S_A\Psi = 0$ and another ten equations from $\overline{S}_A\Psi = 0$. These equations are simply the bosonic expressions associated with each fermionic terms $\psi^A, \chi^A, \psi^A\chi^C\chi_C$, etc, and each bosonic expression therefore equated to zero.

Among the equations derived from $S_A\Psi = 0$ we obtain

$$-\frac{a}{2\sqrt{6}}\frac{\partial A}{\partial a} - \sqrt{\frac{3}{2}}\sigma^2 a^2 A = 0, \tag{22}$$

$$-\frac{\sqrt{2}}{3}\frac{\partial A}{\partial f} + \frac{1}{8\sqrt{2}}\left[1 - (f-1)^2\right]\sigma^2 \ A = 0 \ . \tag{23}$$

These equations correspond, respectively, to the terms linear in $\psi_A, \bar{\lambda}^{(a)}_A$. Eq. (22) and (23) give the dependence of A on a and f, respectively. Solving these equations leads to $A = \hat{A}(a)\tilde{A}(a)$ as

$$A = \tilde{A}(f)e^{-3\sigma^2 a^2}, \tag{24}$$

$$A = \hat{A}(a)e^{\frac{3}{16}\sigma^2\left(-\frac{f^3}{3} + f^2\right)}, \tag{25}$$

A similar relation exists for the $\bar{S}_A\Psi = 0$ equations, which from the $\psi^A\lambda^{(1)}_E\lambda^{(1)E}\lambda^{(2)}_E\lambda^{(2)E}\lambda^{(3)}_E\lambda^{(3)E}$ term in Ψ give for $G = \hat{G}(a)\tilde{G}(f)$

$$G = \tilde{G}(f)e^{3\sigma^2 a^2}, \tag{26}$$

$$G = \hat{G}(a)e^{\frac{3}{16}\sigma^2\left(\frac{f^3}{3} - f^2\right)}. \tag{27}$$

We notice that in our case study, differently to the case of ref. [4, 9, 20]-[26], we are indeed allowed to completely determine the dependence of A and G with respect to the scale factor a and (the *effective* conformal scalar field) f.

The solution (26), (27) is included in the Hartle-Hawking (no-boundary) solutions of ref. [36], where a $k = 1$ FRW universe with Yang-Mills fields was employed within a *non*-supersymmetric

6

quantum cosmologucal point of view. In fact, we basically recover solution (3.8a) in ref. [36] if we replace $f \to f + 1$. As it can be checked, this procedure constitutes the rightful choice according to the definitions employed in [39] for $A^{(a)}_\mu$. Solution (26), (27) is also associated with an anti-self-dual solution of the Euclidianized equations of motion (cf. ref. [36, 38]). However, it is relevant to emphasize that not all the solutions present in [36] can be recovered here. In particular, the Gaussian wave function (26), (27), peaked around $f = 1$ (after implementing the above transformation), represents only one of the components of the wave function in ref. [36]. The wave function in ref. [36] is peaked around the two minima of the corresponding quartic potential. In our model, the potential terms correspond to a "square-root" of the potential present in [36].

Solution (24), (25) has the features of a (Hawking-Page) wormhole solution for Yang-Mills fields [34, 38], which nevertheless has not yet been found in ordinary quantum cosmology. However, in spite of (24), (25) being regular for $a \to 0$ and damped for $a \to \infty$, it may not be well behaved when $f \to -\infty$.

The equations obtained from the cubic and 5-order fermionic terms in $S_A \Psi = 0$ and $\bar{S}_A \Psi = 0$ can be dealt with by multiplying them by $n_{EE'}$ and using the relation $n_{EE'} n^{EA'} = \frac{1}{2} \epsilon_{E'}^{A'}$ Notice that the σ_a matrices are linear independent and are orthogonal to the n matrix. We would see that such equations provide the a, f-dependence of the remaining terms in Ψ. It is important to point out that the dependence of the coefficients in Ψ corresponding to cubic fermionic terms on a and $\phi, \bar{\phi}$ is *mixed* throughout several equations [4, 9]. However, in the present FRW minisuperspace with vector fields, the analogous dependence in a, f occurs in *separate* equations. The equations for cubic and 5-order fermionic terms further imply that any possible solutions are neither the Hartle-Hawking or a wormhole state. In fact, we would get $d_{(a)} \sim a^5 \hat{d}_{(a)}(a) \tilde{d}_{(a)}(f)$ and similar expressions for the other coefficients in Ψ, with a prefactor a^n, $n \neq 0$. This behaviour has also been found in [4]. Hence, from their a-dependence equations these cannot be either a Hartle-Hawking or wormhole state. They correspond to other type of solutions which could be obtained from the corresponding Wheeler-DeWitt equation but with completely different boundary conditions.

Finally, it is righteous to notice that the Dirac bracket of the supersymmetry constraints (18), (19) induces an expression whose bosonic sector corresponds to the gravitational and vector field components of the Hamiltonian constraint present in ref. [36]. Hence, our results (26), (27) are consistent (as expected) within the context of N=1 supergravity being a square-root of gravity [1, 29, 30].

3   Discussions and Conclusions

Summarizing our work, we considered in this paper the canonical formulation of the more general theory of $N = 1$ supergravity with supermatter [26, 35] subject to a $k = +1$ FRW geometry. Our field variables were the graviton and gravitino fields, a vector field $A^{(a)}_\mu$ and corresponding fermionic partners (see the Appendix for further comments). We set the scalar fields and their supersymmetric partners equal to zero.

We derived in section 2 the constraints for our minisuperspace model and solved the Lorentz and supersymmetry constraints. We then obtained non-trivial solutions. We found expressions that can be interpreted as corresponding to a wormhole (Hawking-Page) [34] and (Hartle-Hawking) no-boundary [33] solutions, respectively. The general wave function constructed in the way mentioned in the previous section accomodates naturally the expectation that the early universe — earlier than an inflationary stage — might be dominated by radiation and associated fermionic fields. Our results constitute an approach towards such a supersymmetric scenario.

The Hartle-Hawking solution found here is present in the set of solutions obtained from a Wheeler-DeWitt equation in *non*-supersymmetric quantum cosmology (cf. ref. [36]). That is consistent with our expectations, since N=1 supergravity is a square root of gravity. Moreover, the Dirac bracket of the supersymmetry constraints (18), (19) induces an expression whose bosonic sector is precisely the gravitational and vector field components of the Hamiltonian constraint present in ref. [36]. Hence, the fact that our constraints satisfy a supersymmetry algebra and, in addition, that the solutions of the supersymmetry constraints are consistent with our expectations of canonical formulation adequately supports the choices for the field variables configurations.

As far as the problem of the null result in ref. [21, 22] is concerned, we hope our results may provide a new perspective on this issue. In the least, we know from the present paper that physical states in FRW models with vector fields obtained from N=1 supergravity with supermatter indeed exist. Physical states also exist when solely scalar multiplets are concerned [30]. Thus, we could expect to merge both situations and hopefully obtain non-trivial states. It should be noticed however that so far no analytical solution has been found in non-supersymmetric FRW quantum cosmologies with vector and scalar fields. We hope to address all these issues in a future investigation.

In conclusion, we believe that the results presented here positively add and contribute to our understanding of supersymmetric quantum cosmology. In particular, we hope this paper will further motivate other inquisitive researchers, who would subsequently ameliorate current views on the subject with additonal perspectives.

Supersymmetric quantum cosmology unquestionably constitutes an active and challenging subject for further research (see ref. [30] for an outlook on potential projects). Interesting issues which remain open and we are aiming to address are the following:

a) Obtain conserved currents from Ψ, as consequence of the Dirac-like structure of the supersymmetry constraints [49];

b) Test the validity of minisuperspace approximation in supersymmetric quantum cosmology;

c) Perform the canonical quantization of black-holes in N=2 supergravity.

A   FRW variables for pure N=1 supergravity

In this Appendix we will discuss the choice made for the field variables configuration of our FRW model. In particular, we will analyse how the the ansätze (1), (2), (4), (5) are affected a combination of local coordinate , Lorentz, gauge and supersymmetry transformations.

The ansatz for the tetrad and gravitinos are present in eq. (1), (2). Notice that tetrad and gravitino are not affected by the action of our internal transformation $SU(2)$ when no scalar fields

and fermionic partners are present. Using the expressions (25.14) and (25.15) in ref. [35] or (29)-(33), (39)-(42) and (34)-(35), (36)-(38) in [30] we can obtain (see also ref. [8, 9]) for the tetrad

$$
\begin{aligned}
\delta e^{AA'}{}_i &= \left(-N^{AB} + a^{-1}\xi^{AB} + i\epsilon^{(A}\psi^{B)}\right) e_B{}^{A'}{}_i \\
&+ \left(-\bar{N}^{A'B'} + a^{-1}\bar{\xi}^{A'B'} + i\bar{\epsilon}^{(A'}\bar{\psi}^{B')}\right) e^A{}_{B'i} + \frac{i}{2}\left(\epsilon_C\psi^C + \bar{\epsilon}_{C'}\bar{\psi}^{C'}\right) e^{AA'}{}_i ,
\end{aligned}
\tag{28}
$$

where $\xi^\mu, N^{AB}, \epsilon^A$ are time-dependent vectors or spinors parametrizing local coordinates, Lorentz and supersymmetry transformations. A relation as $\delta e_i^{AA'} = P_1\left[e_\mu^{AA'}, \psi_\mu^A\right] e_i^{AA'}$, where P_1 is an expression (spatially independent and possibly complex) where all spatial and spinorial indices have been contracted, holds provided that the relations

$$
N^{AB} - a^{-1}\xi^{AB} - i\epsilon^{(A}\psi^{B)} = 0 , \quad \bar{N}^{A'B'} - a^{-1}\bar{\xi}^{A'B'} - i\bar{\epsilon}^{(A'}\bar{\psi}^{B')} = 0 ,
\tag{29}
$$

between the generators of Lorentz, coordinate and supersymmetry transformations are satisfied. Hence, we will achieve $\delta e_i^{AA'} = C(t)\delta e_i^{AA'}$ with $C(t) = \frac{i}{2}\left(\epsilon_C\psi^C + \bar{\epsilon}_{C'}\bar{\psi}^{C'}\right)$ and the ansatz (1) will be transformed into a similar configuration. Notice that any Grassman-algebra-valued field can be decomposed into a "body" or component along unity (which takes values in the field of real or complex numbers) and a "soul" which is nilpotent (see ref. [45]). The combined variation above implies that $\delta e^{AA'}{}_i$ exists entirely in the nilpotent ("soul") part.

Let us now address how configuration (2) is transformed. Under local coordinate, Lorentz and supersymmetry transformations (when $\phi = \bar{\phi} = 0$) we get

$$
\begin{aligned}
\delta\psi^A{}_i &= a^{-1}\bar{\xi}^{A'B'} e^A{}_{B'i}\bar{\psi}_{A'} + \frac{3i}{4}\epsilon^A\psi^B\bar{\psi}^{B'} e_{BB'i} - \frac{3i}{8}\epsilon^A\lambda^{(a)C} e_{iCC'}\bar{\lambda}^{(a)C'} - \frac{3i}{4}\epsilon_C\lambda^{(a)C} e^A{}_{C'i}\bar{\lambda}^{(a)C'} \\
&+ \left[2\left(\frac{\dot{a}}{aN} + \frac{i}{a}\right) - \frac{i}{2N}\left(\psi_F\psi^F{}_0 + \bar{\psi}_{F'0}\bar{\psi}^{F'}\right)\right] n_{BA'} e^{AA'}{}_i\epsilon^B .
\end{aligned}
\tag{30}
$$

Hence, to recover a relation like $\delta\psi_i^A = P_2\left[e_\mu^{AA'}, \psi_\mu^A\right] e_i^{AA'}\bar{\psi}_{A'}$, we require $\bar{\xi}^{A'B'} = \xi^{AB} = 0$ (see ref. [9]). In addition, equating the second and third terms in (30) to zero gives, respectively, the contribution of the spin-$\frac{1}{2}$ ψ and λ-fields to the Lorentz constraint. We further need to consider the term $\epsilon_C\lambda^{(a)C} e^A{}_{C'i}\bar{\lambda}^{(a)C'}$ as representing a field variable with indices A and i, for each value of (a). Notice that to preserve the ansatz (2) (see also ref. [9]) we had to require $n_{AB'}\epsilon^B \sim \bar{\psi}_{B'}$ which is not quite $\bar{\psi}$. Here we have to deal with the $\lambda, \bar{\lambda}$–fields and a similar step is necessary. Finally, we get

$$
\left[2\left(\frac{\dot{a}}{aN} + \frac{i}{a}\right) - \frac{i}{2N}\left(\psi_F\psi^F{}_0 + \bar{\psi}_{F'0}\bar{\psi}^{F'}\right)\right] n_{BA'}\epsilon^B \sim P_2\left[e_\mu^{AA'}, \psi_\mu^A\right]\bar{\psi}_{A'}.
\tag{31}
$$

This means that the variation $\delta\psi_i^A = D(t)\psi_i^A$ with $D(t) = \left[2\left(\frac{\dot{a}}{aN} + \frac{i}{a}\right) - \frac{i}{2N}\left(\psi_F\psi^F{}_0 + \bar{\psi}_{F'0}\bar{\psi}^{F'}\right)\right]$ will have a component along unity ("body" of Grassman algebra) and another which is nilpotent (the "soul": $\psi_F\psi^F{}_0 + \bar{\psi}_{F'0}\bar{\psi}^{F'}$).

Concerning the choice (4) for $A_\mu^{(a)}$ it should be noticed that only non-Abelian spin-1 fields can exist consistently within a $k = +1$ FRW background (see ref. [36]-[40]), whose isometry group is $SO(4)$. More specifically, since the physical observables are to be $SO(4)$-invariant, the fields with gauge degrees of freedom may transform under $SO(4)$ if these transformations can be compensated by a gauge transformation. The idea behind the ansatz (4) is to define a homorphism of the isotropy group $SO(3)$ to the gauge group. This homomorphism defines the gauge transformation

which, for the symmetric fields, compensates the action of a given $SO(3)$ rotation. The spin-1 field components in the basis $\left(E_{\hat{c}}^i dx^{\hat{c}}, \tau_{(a)}\right)$ can be expressed as

$$A_i^{(a)} = \frac{f}{2}\delta_i^{(a)} \ . \tag{32}$$

The local coordinate and Lorentz transformations will correspond to isometries and local rotations and these have been compensated by gauge transformations (cf. ref. [36]-[40] for more details). Under supersymmetry transformations we get

$$\delta_{(s)}A_i^{(a)} = iaE_i^b\sigma_{bAA'}\left(\epsilon^A\bar{\lambda}^{A'(a)} - \lambda^{A(a)}\bar{\epsilon}^{A'}\right) \ . \tag{33}$$

Hence, we need to impose the following condition[3]

$$\left\{\begin{array}{rcl} \sigma_{bAA'}\left(\epsilon^A\bar{\lambda}^{A'(a)} - \lambda^{A(a)}\bar{\epsilon}^{A'}\right) &=& E(t) \ , \quad \leftarrow \left\{\begin{array}{l}(a) = b = 1\\(a) = b = 2\\(a) = b = 3\end{array}\right. \\ \sigma_{bAA'}\left(\epsilon^A\bar{\lambda}^{A'(a)} - \lambda^{A(a)}\bar{\epsilon}^{A'}\right) &=& 0 \ , \quad \leftarrow (a) \neq b\end{array}\right. \tag{34}$$

where $E(t)$ is spatially independent and possibly complex, in order to obtain $\delta A_{(a)}^i = P_3\left[e_{AA'\mu}, \psi_\mu^A, A_\mu^{(a)}, \lambda_A^{(a)}\right]A_{(a)}^i$. From eq. (34) it follows that the preservation of the ansatz (4) will require $\delta A_i^{(a)}$ to include a nilpotent ("soul") component. This consequence is similar to the one for the tetrad.

As far as the λ−fields are concerned, we obtain the following result for a combined local coordinate, Lorentz, supersymmetry and gauge transformation:

$$\begin{aligned}\delta\lambda_A^{(a)} = &-\frac{1}{2}\mathcal{F}_{0i}^{(a)}e_A^{A'i}n_{A'}^B\epsilon_B + \frac{i}{2}\mathcal{F}_{ij}^{(a)}\varepsilon_{ijk}h^{\frac{1}{2}}n_{AA'}e^{BA'k}\epsilon_B\\ &-\frac{i}{4}\psi_{A0}\lambda^{(a)A'}n_{A'}^B\epsilon_B - \frac{i}{8}\psi_0^C n_{CC'}\bar{\lambda}^{(a)C'}\epsilon_A - \frac{i}{4}\bar{\psi}_0^{A'}\lambda_A^{(a)}n_{A'}^B\epsilon_B + \frac{i}{8}\bar{\psi}_{C'0}n^{CC'}\lambda_C^{(a)}\epsilon_A\\ &-\frac{i}{4}\bar{\psi}^{A'}n_{AE'}\bar{\lambda}^{(a)E'}n_{A'}^B\epsilon_B - \frac{i}{16}\bar{\psi}_{E'}\bar{\lambda}^{(a)E'}\epsilon_A + \kappa^{abc}\zeta^b\lambda_A^{(c)}\\ &-i\bar{\psi}_{E'}n^{BE'}\bar{\lambda}^{(a)A'}n_{AA'}\epsilon_B + i\bar{\psi}_{E'}\bar{\lambda}^{(a)E'}\epsilon_A - \frac{1}{2}\psi^F\lambda_A^{(a)}\epsilon_F - \frac{i}{\sqrt{2}}\psi^C\lambda_C^{(a)}\epsilon_A\\ &+\frac{i}{8}\psi_A\lambda_E^{(a)}\epsilon^A - \frac{i}{16}\psi_C\lambda^{(a)C}\epsilon_A + 2\epsilon_A\lambda_B^{(a)}\psi^B + \epsilon_C\psi^C\lambda_A^{(a)} \ ,\end{aligned} \tag{35}$$

where

$$\mathcal{F}_{0i}^{(a)} = \dot{f}\delta_i^{(a)} \ , \ \mathcal{F}_{ij}^{(a)} = \frac{1}{4}(2f - f^2)\varepsilon_{ij(a)} \ . \tag{36}$$

The last sixth terms in eq. (35) may be put in a more suitable form in order to obtain $\delta\lambda_{(a)}^A = P_4\left[e_{AA'\mu}, \psi_\mu^A, A_\mu^{(a)}, \lambda_A^{(a)}\right]\lambda_{(a)}^A$. This would require that the remaining terms to satisfy a further condition equated to zero.

The above results concerning the transformations of the physical variables are consistent with a FRW geometry if the mentioned restrictions are provided. One may ask if these new relations that have to be provided will themselves be invariant under a supersymmetry transformation or even a combination of supersymmetry, Lorentz and other transformations. Using these relevant

---

[3]If we had chosen $\lambda_A^{(a)} = \lambda_A$ for any value of (a) then we would not be able to obtain a consistent relation similar to (34). Namely, such that $\delta A_1^{(1)} \sim A_1^{(1)}$ and $\delta A_1^{(2)} \sim A_1^{(2)} = 0$.

transformations we just obtain *additional* conditions for the previous ones to be invariant. We can proceed with this process within a recursive way but neither a contradiction or a clear indication that the relations are invariant is produced.

However, supersymmetry will be one of the features of our model, in spite of the new relations that will be produced. In fact, the constraints of our FRW model satisfy a supersymmetry algebra, i.e., we can see from (14), (18), (19), (20) that $[S_A, \bar{S}_B]_D \sim H + J_{AB}$, which is fully consistent with supersymmetry. In addition, the solutions obtained by solving the equations $S_A \Psi = 0$ and its Hermitian conjugate are in agreement with what it should be expected with N=1 supergravity being a square-root of gravity. Namely, our solutions (26), (27) are also present in the set of solutions found in ref. [39] for the case of a *non*-supersymmetric quantum FRW model with Yang-Mills fields.

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